

## SOME SAMPLE MATERIALS from <https://physport.org/curricula/ACEQM>

Our full download provides notes and suggestions from large lecture settings at CU Boulder and smaller class settings at Cal Poly Pomona and Cal State University Fullerton.

Materials are organized by topic (following the presentation in McIntyre's textbook)

Below, just a few samples to get a flavor for the sorts of materials you will find.

We ask that you make an effort to personalize any of these material you use to match your own goals and population, as well as to avoid having materials (and solutions!) too easily available online.

For more explanation, also see our AJP article: "Adaptable research-based materials for teaching quantum mechanics", S. Pollock, G. Passante, H. Sadaghiani, *Am. J. Phys.* 91, 40–47 (2023)

<https://doi.org/10.1119/5.0109124>

### Pre-class questions

1. Consider the following equations:

$$\hat{A}|a_1\rangle = 5\text{eV}|a_1\rangle \quad \hat{A}|a_2\rangle = -4\text{eV}|a_2\rangle$$

- a) Label each component of those expressions (i.e.,  $\hat{A}$ ,  $|a_1\rangle$ , 5 eV) with the appropriate quantum mechanical and/or mathematical term(s).
- b) Express  $\hat{A}$  as a 2 x 2 matrix in the orthonormal  $\{|a_1\rangle, |a_2\rangle\}$  basis.

2. Now consider the following expressions:

$$|b_1\rangle = \frac{1}{\sqrt{3}}|a_1\rangle + \frac{\sqrt{2}}{\sqrt{3}}|a_2\rangle \quad |b_2\rangle = \frac{\sqrt{2}}{\sqrt{3}}|a_1\rangle + \frac{1}{\sqrt{3}}|a_2\rangle$$

where  $\hat{B}|b_1\rangle = 2\text{eV}|b_1\rangle$  and  $\hat{B}|b_2\rangle = 7\text{eV}|b_2\rangle$ .

- a) Starting with a particle in the state  $|b_1\rangle$ , if you measure  $\hat{A}$ , what value(s) could you obtain, and with what probabilities?
- b) After this measurement of  $\hat{A}$ , if you subsequently measure  $\hat{B}$ , what value(s) could you get?

### Clicker questions (conceptual question asked in class)

What does it mean if an operator is on the right of a ket?

To be specific, what kind of object is  $|\psi\rangle\hat{A}$ ?

- A. A ket  
B. A bra  
C. An operator  
D. This is ill-defined (correct)

Given the quantum state:  $|\psi\rangle = \frac{1}{2}|+\rangle + e^{i\pi/4}\frac{\sqrt{3}}{2}|-\rangle$

What is the probability that a measurement of  $\hat{S}_z$  will yield the value  $-\hbar/2$ ?

- A. 0.25  
B.  $\sqrt{3}/2$   
C. 0.75 (correct)  
D. Something else

## Part of our “quantum mouse” tutorial

(Where we have already introduced novel kets of “happy” or “sad” mice, or in another basis, “wide eyed” or “closed eyed”)

2. Suppose I now tell you:  $|\odot\rangle = \frac{1}{\sqrt{5}}|\oplus\rangle + \frac{2}{\sqrt{5}}|\ominus\rangle$

- This suggests that wide-eyed quantum mice are rather *stressed*. Briefly, why might I say that? (To consider: could a quantum mouse have their mood measured to be “stressed”?)
  - Use orthonormality and completeness to expand the  $S = 1$  mm eigenstate in the mood basis. i.e., in the expression  $|*\rangle = a|\oplus\rangle + b|\ominus\rangle$ , I’m asking you to find the numbers  $a$  and  $b$ .
  - Is your answer for  $a$  and  $b$  unique? (If not, does it matter?)
3. Let’s see some consequences of the above assumptions. In this curious quantum world, suppose I give you a mouse, you measure  $S$  and get the eigenvalue 1 mm.
- What quantum state is the mouse in now? Is there any ambiguity about the *state* at this point?
  - If you now re-measure  $S$  on this state, what result(s) can you get, with what probabilities? After this second measurement of  $S$ , what state will you be in?
  - Following the above measurement, what is the probability that a subsequent measurement of  $M$  will yield a result of -1 (i.e., unhappy)? Explain. (*Don’t intuit answers at this point, work it out from the postulates of quantum mechanics!*)
  - I said above “wide-eyed mice are rather stressed”. In the same spirit, how might you describe small-eyed mice?

## Homework question

A beam of spin-1/2 silver atoms travel along the  $+z$ -axis, passing through a pair of Stern-Gerlach analyzers, each analyzing the  $y$ -component of spin. The first analyzer allows only particles with spin up (along  $y$ ) to pass. The second analyzer allows only particles with spin down (along  $y$ ) to pass. The atoms travel at speed  $v$  between the analyzers, which are separated by a region of variable length  $d$ , in which there is a uniform magnetic field  $B_0\hat{z}$ . Define  $t = 0$  as the moment an atom passes the first analyzer.

- Sketch the setup. If  $B_0 = 0$ , how many atoms pass the 2<sup>nd</sup> analyzer? Why?
- Calculate the spin state  $|\psi(t)\rangle$  when an atom is between the analyzers.
- Given  $B_0$ , what is the probability a given atom passes the second analyzer? Sketch this probability as a function of distance  $d$  between the analyzers. Does your result at  $d = 0$  make sense?
- If  $B_0 > 0$ , find the smallest distance  $d$  such that 50% of the atoms transmitted by the first analyzer pass the second analyzer.
- Explain physically why more particles pass in part d than part a. If  $B_0$  had been much stronger, would your answer to part d (the smallest distance) be bigger, smaller, or the same? Discuss!

### Exam question:

Consider Schrödinger's quantum mechanical cat. You can measure the tail length in cm using the operator  $\hat{T}$ . There are only two possible measurement outcomes for the tail length as shown in the eigenequations:

You can also measure something quite different: cat mood (with associated operator  $\hat{M}$ ). Mood is a dimensionless quantity, with value +1 for a happy cat, and -1 for a sad cat. The eigenequations are:

Both “tail length” and “mood” eigenstates form complete, orthonormal bases for cats.

Finally, suppose that a “happy cat” eigenstate can be written in terms of the tail length basis as:

- a) What measured outcome(s) can I get, with what probabilities, if I measure the tail length of a happy cat?
- b) What is the probability that a short-tailed cat will be measured to be “sad” (i.e., -1)? Show/explain your work.
- c) If you know that the expectation value for the cat's mood is 0.25, what is the probability that the cat would be measured to be “happy” (i.e., +1)?

**Downloads also include:** notes for faculty on all of the above, faculty-consensus course learning goals, sample lecture slides, our end-of-term research-based conceptual assessment (“QMCA”), and more.